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10MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The equation solvable for p is of the form
A) $x = f(y, p)$ B) $y = f(x, y)$ C) $p = f(x, y)$ D) $x = \phi_1(p, c)$

ii) The general solution of first order and higher degree differential equation contains
A) Three constants B) Four constants C) Two Constants D) One constant

iii) An equation of the form $y = p_x + f(p)$ is known as
A) Clairaut's forms B) Solvable for p C) Solvable for y D) Solvable for x

iv) The algebraic sum of the voltage drop across each closed circuit is equal to
A) Constant B) Ri C) emf D) 2.

b. Solve : $x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} = 6y^2 = 0$. **(06 Marks)**

c. Solve : $y = 2px + p^m$. **(05 Marks)**

d. Solve : $(px - y)(py + x) = 2p$. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The complementary function in the solution of differential equation depends upon
A) Nature of the roots B) Order of the equation
C) Degree of the equation D) None of these

ii) In two simultaneous differential equation the number of independent variables are
A) One B) Two C) Three D) More than 3

iii) The initial conditions are imposed at the
A) Different points B) Different end points
C) Same point D) None of these

iv) If the RHS of differential equation is e^{ax}, then particular integral is
A) $\frac{1}{f(D^2)}e^{ax}$ B) $\frac{x}{f(D)}e^{ax}$ C) $\frac{1}{f(a)}e^{ax}$ D) $\frac{1}{f'(D)}e^{ax}$.

b. Solve : $(D^2 - 4D + 3)y = e^{2x}\sin 3x$. **(06 Marks)**

c. Solve : $e^x\left(\frac{d^2y}{dx^2}\right) + 2e^x\frac{dy}{dx} + e^x y = x^2$. **(05 Marks)**

d. Solve : $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$. **(05 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) Method of variation of parameter is used to find _____ of the differential equation.
   A) Complementary function B) Particular integral
   C) Integration D) None of these

ii) The Wronskian of the complementary solutions u = cos 2x, v = sin 2x is
   A) 4 B) 2 C) 3 D) 8

iii) By substituting $x = e^z$ Cauchy's differential equation reduces to
   A) Simultaneous equations
   B) Non – linear equations
   C) Algebraic equation
   D) Differential equation with constant coefficient

iv) In Frobenius method solution is assumed to be
   A) Finite convergent series B) Infinite convergent series
   C) Infinite divergent series D) Alternative series.

b. Find one particular solution of :

$$9x(1-x)\frac{d^2y}{dx^2}-12\frac{dy}{dx}+4y=0$$

By Frobenius method. **(06 Marks)**

c. Solve $y''+y=\tan x$ by using the method of variation of parameters. **(05 Marks)**

d. Solve $(2x+3)^2\frac{d^2y}{dx^2}-2(2x+3)\frac{dy}{dx}-12y=6x$. **(05 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) In the partial differential equation number of dependent variable
   A) One B) Two C) Three D) Four

ii) Solution of $\frac{\partial^2 z}{\partial y^2}=\sin y$ is
   A) $z=\frac{x^3}{6}y+xf(y)$ B) $z=c_1e^y+c_2e^{-y}$
   C) $z=\sin y+yf(x)+g(x)$ D) $z=e^y\cos x+\sin x$

iii) One set of multipliers for the equations : $\frac{dx}{x(y-x)}=\frac{dx}{y(z-x)}=\frac{dz}{z(x-y)}$ are
   A) 1, 1, 1 B) 1, 2, 3 C) x, 2, z D) $\frac{1}{x}, y, \frac{1}{z}$

iv) The solution of $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$ to be assumed in method of separation of variable is
   A) Z = X(y) Y(x) B) Z = X(x) Y(z)
   C) Z= X(x) Y(y) D) $Z=e^x f(y)$.

b. Solve : $(x^2-y^2-z^2)p+2xyq=2xz$. **(06 Marks)**

c. Form the partial differential equation by eliminating the function from

$$Z=y^2+2f\left(\frac{1}{x}+\log y\right).$$ **(05 Marks)**

d. Solve by the method of separation of variables :

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0.$$ **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) Changing the order of integration leads to change of
A) Limits B) Area C) Volume D) Length

ii) If $x = r\cos\theta$, $y = r\sin\theta$ and $|J| = r$, then $\iint_R f(x,y)dxdy$ is

A) $\iint_R f(r,\theta)r\,dr\,d\theta$ B) $\iint_R f(r,\theta)dr\,d\theta$ C) $\iint_R f(r,\theta)dx\,dy$ D) $\iint_R f(x,y)r\,dr\,d\theta$

iii) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ is

A) $\sqrt{\pi}$ B) π C) $\pi\sqrt{\pi}$ D) 2π

iv) If m,n are the integers then $\beta(m, n)$ is defined as

A) $\int_{-\infty}^{\infty} x^m(1-x)^n dx$ B) $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ C) $\int_{-1}^{1} x^m(1-x)^n dx$ D) $\int_{-\infty}^{\infty} x^m(1-x)^{n-1}dx$

b. Evaluate $\iint_A y\,dx\,dy$, where A is the region enclosed between the parabola $y^2 = 4x$ and $x^2 = 4y$. **(06 Marks)**

c. Evaluate : $\int_0^3\int_0^2\int_0^1 (x+y+z)dz\,dx\,dy$. **(05 Marks)**

d. Evaluate : $\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta \times \int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}$. **(05 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If the initial and terminal points are same, then the curve is said to be
A) Smooth B) Closed C) Open D) Breaking

ii) The total work done by force $\vec{F}$ during displacement from A to B is given by

A) $\int_B^A \vec{F}\cdot d\vec{R}$ B) $\int_a^b \vec{F}\cdot d\vec{R}$ C) $\int_A^B \vec{F}\cdot d\vec{R}$ D) $\int \vec{F}\cdot d\vec{R}$

iii) If $\vec{V}$ is velocity of the fluid particle, C is the closed curve and the integral $\oint \vec{V}\cdot d\vec{R} = 0$, the $\vec{V}$ is said to be
A) Irrotational B) Solenoidal C) Rotational D) Force

iv) For any closed surface S the value of $\iint_S \text{curl}\,\vec{F}\cdot\vec{N}\,dS =$ ______

A) 1 B) Zero C) 2 D) 4.

b. A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$ evaluate the line integral over the circular path given by $x^2 + y^2 = a^2$, $z = 0$. **(06 Marks)**

c. Verify Green's theorem for $\int_C [(xy+y^2)dx + x^2dy]$ where C is bounded by $y = x$ and $y = x^2$. **(05 Marks)**

d. Evaluate $\int_S \vec{F}\cdot dS$ where $\vec{F} = 4x\,\hat{i} - 2y^2\,\hat{j} + z^2\,\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(05 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) If $L\{f(t)\} = F(s)$, then $L\{e^{at} f(t)\} =$ _____

A) F(s) B) F(s – b) C) F(s – a) D) SF(s) – f(0)

ii) If $L\{f(t)\} = F(s)$, then $L\{f(at)\} =$ _____

A) $F(s/a)$ B) $\frac{1}{a} F(s/a)$ C) $\frac{1}{S} F(s/a)$ D) f(s) = F(0)

iii) The Laplace transform of periodic function with period T is

A) $\int_0^T e^{-St} f(t) dT$ B) $\frac{1}{1-e^{-ST}} \int_0^a e^{-ST} f(t) dt$ C) $\frac{1}{1-e^{-ST}} \int_0^T e^{-ST} f(t) dt$ D) $\frac{1}{1-e^{-at}} \int_0^T e^{-ST} f(t) dt$

iv) Laplace transform of unit step function H(t – a) is

A) $\frac{1}{S} F(s)$ B) $\frac{1}{S} e^{-as}$ C) $\frac{1}{S}$ D) $\frac{1}{S^2} F(s)$.

b. Find the Laplace transform of the following : i) $e^{-3t} \cos^3 2t$ ii) $t \cos at$. (06 Marks)

c. Find the Laplace transform of a periodic function with period a is given by

$$f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}.$$ (05 Marks)

d. Expression f(t) in terms of Heaviside's function and find its Laplace transform

$$f(t) = \begin{cases} \sin t & o < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t > 2t \end{cases}.$$ (05 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) If $L\{f(t)\} = F(s)$, then $L^{-1}\{F(s)\} =$ _____

A) $L\{f(t)\}$ B) $f'(t)$ C) f(t) D) $\int_0 e^{-st} f(t) dt$

ii) Inverse Laplace transform of $\frac{1}{(s-a)^2}$ is

A) e^{at} B) e^{-at} C) $\frac{e^{-at}}{t}$ D) $e^{at} t$

iii) Inverse Laplace transform of $\frac{s^3 + s^2 + 6}{s^4}$ is

A) $1 + t + t^3$ B) $2 + 3 + 4t^4$ C) $t + t^2 + 3t^3$ D) $\frac{1+t^3}{t}$

iv) If $L\{f(t)\} = F(s)$ and $L\{g(t)\} = G(s)$, then $L^{-1}\{F(s)\} \cdot G(s) =$ ____

A) $\int_0^\infty f(u) g(t-u) du$ B) $\int_0^t f(u) g(t-u) du$ C) $\int_0^t f(t) g(u-t) dt$ D) $\int_0^\infty f(u) g(t-u) du$.

b. Solve by the method of Laplace transforms the equation :

$\frac{d^2u}{dt^2} - 2\frac{dy}{dt} + y = e^t$ give $y(0) = 2$, $y'(0) = -1$. (06 Marks)

c. Find the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. (05 Marks)

d. Evaluate $L^{-1}\left\{\frac{1}{(s-4)(s+5)}\right\}$ using convolution theorem. (05 Marks)

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